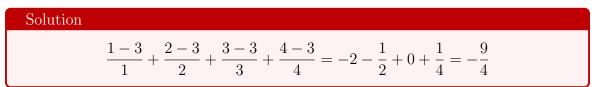
1. Write the sum  $\sum_{i=1}^{4} \frac{i-3}{i}$  without sigma notation, then evaluate it.



- 2. Approximate the area under the graph of  $f(x) = 2x^2$  and above the x-axis from x = 0 to x = 4 using n = 4 and the following methods.
  - (a) Using left endpoints.

Solution Since n = 4 then  $\Delta x = \frac{4-0}{4} = 1$ , thus the intervals are [0, 1], [1, 2], [2, 3], [3, 4]. Using left endpoints, the approximation is  $f(0) \cdot 1 + f(1) \cdot 1 + f(2) \cdot 1 + f(3) \cdot 1 = 28$ 

(b) Using right endpoints.

Solution

Using the same intervals as above, we have the right endpoint approximation is:

$$f(1) + f(2) + f(3) + f(4) = 60$$

(c) Using the midpoints.

Solution

Using the same intervals as above, we have the midpoint approximation is:

$$f(\frac{1}{2}) + f(\frac{3}{2}) + f(\frac{5}{2}) + f(\frac{7}{2}) = 42$$

3. Express the limit as a definite integral on the interval [2, 5]. (Do not solve the integral).

$$\lim_{n \to \infty} \sum_{i=1}^n \sqrt{x_i^2 - 2\Delta x}$$

Solution  $\int_{2}^{5} \sqrt{x^{2} - 2} dx$ 

4. Suppose that f and g are integrable. Find the integrals below using the following information:

$$\int_{1}^{9} f(x)dx = -2, \ \int_{7}^{9} f(x)dx = 10, \ \int_{7}^{9} g(x)dx = 3$$

(a)  $\int_{9}^{7} [f(x) - g(x)] dx$ 

Solution

$$\int_{9}^{7} [f(x) - g(x)]dx = -\int_{7}^{9} [f(x) - g(x)]dx$$
$$= -[\int_{7}^{9} f(x)dx - \int_{7}^{9} g(x)dx]$$
$$= -\int_{7}^{9} f(x)dx + \int_{7}^{9} g(x)dx$$
$$= -10 + 3 = -7$$

(b)  $\int_1^7 f(x) dx$ 

Solution

 $\int_{1}^{7} f(x)dx = \int_{1}^{9} f(x)dx - \int_{7}^{9} f(x)dx$ = -2 - 3 = -5

(c)  $\int_{7}^{9} [3g(x) + 2f(x)] dx$ 

Solution

$$\int_{7}^{9} [3g(x) + 2f(x)]dx = 3\int_{7}^{9} g(x)dx + 2\int_{7}^{9} f(x)dx$$
$$= 3 \cdot 3 + 2 \cdot 10 = 29$$

5. Evaluate the following integrals.

(a) 
$$\int_{-2}^{2} (x^3 - 2x + 1) dx$$

## Solution

$$\int_{-2}^{2} (x^3 - 2x + 1)dx = \int_{-2}^{2} x^3 dx - \int_{-2}^{2} 2x dx + \int_{-2}^{2} 1dx$$

Notice either by computing the Reimann sum or looking at a graph that  $\int_{-2}^{2} x^{3} dx = 0$  as well as  $\int_{-2}^{2} 2x dx = 0$ . Thus

$$\int_{-2}^{2} (x^3 - 2x + 1)dx = \int_{-2}^{2} 2^2 1dx$$
$$= \lim_{n \to \infty} \sum_{i=1}^{n} 1 \cdot \frac{4}{r}$$
$$= \lim_{n \to \infty} \frac{4}{n} \sum_{i=1}^{n} 1 = 1^n 1$$
$$= \lim_{n \to \infty} \frac{4}{n} \cdot n$$
$$= \lim_{n \to \infty} 4 = 4$$

 $\frac{4}{n}$ 

(b)  $\int_0^4 (3-2x) dx$ 

Solution  

$$\Delta x = \frac{4}{n} = 1 \qquad x_i = 0 + \frac{4}{n}i = \frac{4i}{n}$$

$$\int_0^4 (3 - 2x)dx = \lim_{n \to \infty} \sum_{i=1}^n \left(3 - 2(\frac{4i}{n})\right) \cdot \frac{4}{n}$$

$$= \lim_{n \to \infty} \frac{4}{n} \sum_{i=1}^n (3 - \frac{8i}{n})$$

$$= \lim_{n \to \infty} \frac{4}{n} \left[\sum_{i=1}^n 3 - \sum_i i = 1^n \frac{8i}{n}\right]$$

$$= \lim_{n \to \infty} \frac{4}{n} \left[3n - \frac{8}{n} \sum_i i = 1^n i\right]$$

$$= \lim_{n \to \infty} \frac{4}{n} \left[3n - \frac{8}{n} \sum_i i = 1^n i\right]$$

$$= \lim_{n \to \infty} 12 - \frac{16n(n+1)}{n^2}$$

$$= 12 - 16 = -4$$