1. Use the first part of the Fundamental Theorem of Calculus to find the derivatives of the following functions.

   (a) $g(x) = \int_1^x \sin(3t^2)dt$

   Solution

   $g'(x) = \sin(3x^2)$

   (b) $h(x) = \int_x^0 (u^2 - u)^3 du$

   Solution

$$\begin{aligned} h(x) &= -\int_0^x (u^2-u)^3 du \\ h'(x) &= \frac{d}{dx}\left[-\int_0^x (u^2-u)^3 du\right] \\ &= -\frac{d}{dx}\left[\int_0^x (u^2-u)^3 du\right] \\ &= -(x^2-x)^3 \end{aligned}$$

   (c) $R(x) = \int_1^{x^2} \sqrt{3+t^2}dt$

   Solution

   Let's substitute $u = x^2$.

$$\begin{aligned} R(x) &= \int_1^u \sqrt{3+t^2}dt \\ R'(x) &= \frac{d}{dx}\left[\int_1^u \sqrt{3+t^2}dt\right] \qquad = \frac{d}{du}\left[\int_1^u \sqrt{3+t^2}dt\right] \cdot \frac{du}{dx} \\ &= \sqrt{3+u^2} \cdot 2x \\ R'(x) &= \sqrt{3+x^4} \cdot 2x \end{aligned}$$

2. Evaluate the following integrals.

   (a) $\int_{-6}^3 \pi dt$

Solution

$$\int_{-6}^{3} \pi dt = \pi t \mid_{-6}^{3}$$
$$= 3\pi - (-6\pi)$$
$$= 9\pi$$

(b) $\int_{\frac{-\pi}{4}}^{\frac{\pi}{2}} \sin\theta d\theta$

Solution

$$\int_{\frac{-\pi}{4}}^{\frac{\pi}{2}} \sin\theta d\theta = [-\cos\theta]_{-\frac{\pi}{4}}^{\frac{\pi}{2}}$$
$$= -\cos(\frac{\pi}{2} + \cos(-\frac{\pi}{4})$$
$$= \frac{\sqrt{2}}{2}$$

(c) $\int_{-1}^{4} |2x-3|dx$

Solution

Notice that $|2x-3|$ is a continuous peice-wise function

$$|2x-3| = \begin{cases} 2x-3 & x \geq \frac{3}{2} \\ -(2x-3) & x < \frac{3}{2} \end{cases}$$

So we should evaluate the integral separately for the different peices.

$$\int_{-1}^{4} |2x-3|dx = \int_{-1}^{\frac{3}{2}} |2x-3|dx + \int_{\frac{3}{2}}^{4} |2x-3|dx$$
$$= \int_{-1}^{\frac{3}{2}} -2x+3dx + \int_{\frac{3}{2}}^{4} 2x-3dx$$
$$= \left[-x^2+3x\right]_{-1}^{\frac{3}{2}} + \left[x^2-3x\right]_{\frac{3}{2}}^{4}$$
$$= \frac{9}{4} + \frac{9}{2} - (-1-3) + 16 - 12 - (\frac{9}{4} - \frac{9}{2})$$
$$= \frac{25}{2}$$

(d) $\int u^2 + \frac{u}{4} - 16du$

Solution

$$\int u^2 + \frac{u}{4} - 16du = \frac{u^3}{3} + \frac{u^2}{8} - 16u + C$$

(e) $\int 2 + \sec^2(\theta)d\theta$

Solution

$$\int 2 + \sec^2(\theta)d\theta = 2\theta + \tan\theta + C$$

(f) $\int \sqrt{x^7} - \frac{1}{\sqrt{x}}dx$

Solution

$$\begin{aligned}\int \sqrt{x^7} - \frac{1}{\sqrt{x}}dx &= \int x^{\frac{7}{2}}dx - \int x^{-\frac{1}{2}}dx \\ &= \frac{2x^{\frac{9}{2}}}{9} - 2x^{\frac{1}{2}} + C\end{aligned}$$

(g) $\int 4x\sqrt{1-x^2}dx$

Solution

We need to use substitution. Let $u = 1 - x^2$. Then $du = -2xdx$, so $-2du = 4xdx$. Then we have the following:

$$\begin{aligned}\int 4x\sqrt{1-x^2}dx &= \int \sqrt{u}\cdot(-2)du \\ &= -2\int \sqrt{u}du \\ &= -2\left(\frac{2u^{\frac{3}{2}}}{3}\right) + C \\ &= \frac{-4u^{\frac{3}{2}}}{3} + C \\ &= \frac{-4\sqrt{(1-x^2)^3}}{3} + C\end{aligned}$$

(h) $\int \sec(5\omega)\tan(5\omega)d\omega$

Solution

Let $u = 5\omega$. Then $du = 5d\omega$, and $\frac{1}{5}du = d\omega$.

$$\begin{aligned}\int \sec(5\omega)\tan(5\omega)d\omega &= \int \frac{1}{5}\sec(u)\tan(u)du \\ &= \frac{1}{5}\int \sec(u)\tan(u)du \\ &= \frac{1}{5}\sec(u) + C \\ &= \frac{1}{5}\sec(5\omega) + C\end{aligned}$$

(i) $\int_{-1}^{0}(3t-1)^{20}dt$

Solution

Let $u = 3t - 1$ so $du = 3dt$, and $\frac{1}{3}du = dx$. When $t = 0, u = -1$, and when $t = -1, u = -4$.

$$\begin{aligned}\int_{-1}^{0}(3t-1)^{20}dt &= \int_{-4}^{-1}\frac{1}{3}u^{20}du \\ &= \frac{u^{21}}{3\cdot 21}\Big|_{-4}^{-1} \\ &= \frac{(-1)^{21}}{63} - \frac{(-4)^{21}}{63} \\ &= \frac{-(1+4^{21})}{63}\end{aligned}$$

(j) $\int x(2x+5)^8 dx$

Solution

Let $u = 2x - 5$. Then $x = \frac{u-5}{2}$ and $du = 2dx$, and $\frac{1}{2}du = dx$.

$$\begin{aligned}\int x(2x+5)^8 dx &= \int \frac{u-5}{2} \cdot u^8 \cdot \frac{1}{2} du \\ &= \frac{1}{4}\int u^9 - 5u^8 du \\ &= \frac{1}{4}\left(\frac{u^{10}}{10} - \frac{5u^9}{9}\right) + C\end{aligned}$$

$$\frac{u^{10}}{40} - \frac{5u^9}{36} + C$$

3. The velocity function in m/s is $v(t) = 3 - 4t + t^2$. Find the displacement and distance traveled by the particle during the interval $0 \le t \le 4$.

Solution

The displacement is given by

$$\int_0^4 v(t)dt = \int_0^4 3 - 4t + t^2 dt = \frac{4}{3}$$

To find the distance, we need to calculate the absolute value of the distance traveled. (ie, determine the distance traveled backwards and forwards.)
Notice that $v(t) \ge 0$ if t is in $[0,1] \cup [3,4]$ and $v(t) \le 0$ if t is in $[1,3]$.

$$\begin{aligned}\int_0^4 |v(t)|dt &= \int_0^1 v(t)dt + \int_3^4 v(t)dt + \int_1^3 -v(t)dt \\ &= 4\end{aligned}$$