

1. Sketch the graph of a function on $[-3, 3]$ that has an absolute maximum but no local maximum.

2. Sketch the graph of a function on $[-3, 3]$ that has a local minimum but no absolute minimum .

3. Find the critical numbers of the following functions.

(a) $f(x) = \sqrt{x}(x^2 - 3x)$

(b) $f(x) = 2x^3 - 3x^2 - 36x$

(c) $f(\theta) = 2 \cos \theta + \sin^2 \theta$

4. Find the absolute maximum and absolute minimum values of f on the given interval.

(a) $f(x) = (t^2 - 4)^3, [-2, 3]$

(b) $f(x) = \frac{x}{x^2 - x + 1}$

5. Verify that the function satisfies the 3 hypotheses of Rolle's theorem on the given interval. Then, find all numbers c that satisfy the conclusion of Rolle's Theorem.

$$f(x) = \sin(x/2), \quad [-\pi/2, 3\pi/2]$$

6. Verify that the function satisfies the hypothesis of the Mean Value Theorem on the given interval, and then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

$$3x^2 - 4x + 1, \quad [0, 2]$$

7. Show that a polynomial of degree 3 has at most 3 real roots.

8. Suppose that $f'(x) \leq 2$ for all x in $[1, 5]$. If $f(5) = 10$ what's the largest $f(1)$ could be?

9. Bonus: A number a is called a fixed point of a function f if $f(a) = a$. Prove that if $f'(x) \neq 1$ for all real numbers x then f has at most one fixed point.