

Goal: complete the following problems without using notes or calculators.

1. Calculate the following limits.

$$(a) \lim_{x \rightarrow 0} \frac{\sqrt{3x+4}-2}{x}$$

$$(b) \lim_{x \rightarrow 3} \left( \frac{\sin(3-x)}{x-3} + \frac{3x^2+x-2}{x+1} \right)$$

$$(c) \lim_{x \rightarrow 0} 3x^2 \cos\left(\frac{2}{x}\right)$$

$$(d) \lim_{x \rightarrow 3^-} \frac{|x-3|}{x-3}$$

2. Use the definition of the derivative to find the derivatives of the following functions.

(a)  $f(x) = 3x^2 - 2x + 10$

(b)  $g(x) = \sqrt{3x - 5}$

(c)  $h(x) = \frac{1}{t^2}$

3. If possible, find the equation of the tangent line to the above curves when  $x = 1$ .

4. Find the derivatives of the following functions.

(a)  $f(x) = \cos(\sin(x^2))$

(b)  $y = \cos\left(\frac{x}{2}\right)\sqrt{x^2 + 4}$

(c)  $\frac{3x^2+1}{x-4}$

(d) Find  $\frac{dy}{dx}$  when  $y^2 - 19 = -3xy + x$

5. Show that the function  $f(x) = x^5 - 2x^3 - 2$  has a zero (root/x-intercept) in the interval  $[0, 2]$ . Make sure to state which theorem you're using and why it applies.

6. Show that the function  $f(x) = \frac{x^3 - 2x^2 + 1}{x - 2}$  is 0 for some real value of  $x$ . Make sure to state which theorem you're using and why it applies.

7. Consider the following function.

$$f(x) = \begin{cases} 2x + 3 & x \leq -2 \\ 1 & -2 < x < 0 \\ 0 & x = 0 \\ -x + 4 & x > 0 \end{cases}$$

(a) Graph the function.

(b) Use the graph to determine  $\lim_{x \rightarrow 0} f(x)$ .

(c) Use the graph to determine  $\lim_{x \rightarrow -2^-} f(x)$ .

(d) Determine the intervals on which  $f(x)$  is continuous. (Be clear about endpoints).

(e) Determine the intervals on which  $f(x)$  is differentiable. (Be clear about endpoints).