1. Evaluate the difference quotient for the given function. Simplify your answer.

$$f(x) = x^2 - 4x + 5 \qquad \frac{f(x+h) - f(x)}{h}$$

Solution

$$\begin{aligned}
\frac{f(x+h) - f(x)}{h} &= \frac{((x+h)^2 - 4(x+h) + 5) - (x^2 - 4x + 5)}{h} \\
&= \frac{x^2 + 2xh + h^2 - 4x - 4h + f - x^2 + 4x - 5}{h} \\
&= \frac{2xh + h^2 - 4h}{h} \\
&= 2x + h - 4
\end{aligned}$$

2. Find the domain of the following functions.

   (a) $f(x) = \frac{x+6}{x^2-3x-4}$

   (b) $f(x) = \sqrt{3 - \sqrt{x}}$

   (c) $g(x) = \sqrt[3]{x - 8}$

Solutions

(a) Need to make sure we are not dividing by zero.
$x^2 - 3x - 4 = (x-4)(x+1) = 0$ if $x = 4$ or $x = -1$
Domain: $(-\infty, -1) \cup (-1, 4) \cup (4, \infty)$

(b) Need to make sure we aren't taking the square root of a negative number.
First, we see that $x \geq 0$. Next, we have that

$$\begin{aligned}
3 - \sqrt{x} &\geq 0 \\
3 &\geq \sqrt{x} \\
9 &\geq x
\end{aligned}$$

Domain: $[0, 9]$

(c) Domain: $(-\infty, \infty)$

3. Find $g \circ f$ and $f \circ g$ and their domains.

$$g(x) = \sqrt{x+3} \quad f(x) = x + \frac{1}{x}$$

Solution

$$(f \circ g)(x) = \sqrt{x+3} + \frac{1}{\sqrt{x+3}}$$

For the domain we need to make sure $x + 3 \geq 0$ and $\sqrt{x+3} \neq 0$.
Domain: $(-3, \infty)$

$$(g \circ f)(x) = \sqrt{x + \frac{1}{x} + 3}$$

For the domain we need to make sure we are taking the square root of a non-negative number and not dividing by 0, so we are solving $x + \frac{1}{x} + 3 = \frac{x^2+3x+1}{x} \geq 0$. **This is harder than I intended**. It's okay for $x^2 + 3x + 1$ to be negative as long as x is negative as well. Notice that $x^2 + 3x + 1 \leq 0$ if and only if $\frac{-3-\sqrt{5}}{2} \leq x \leq \frac{-3+\sqrt{5}}{2} \leq 0$. So if x is positive, so is the polynomial, hence we get the following domain.

$$(\frac{-3-\sqrt{5}}{2}, \frac{-3+\sqrt{5}}{2}) \cup (0, \infty)$$

4. Find two functions $f(x)$ and $g(x)$ such that $(f \circ g)(x) = x^2 + 4x + 2$

Possible Solutions

$$f(x) = x + 2 \quad g(x) = x^2 + 4$$

or

$$f(x) = x^2 - 2 \quad g(x) = x + 2$$

5. If a ball is thrown into the air with a velocity of 40 ft/s , its height in feet t seconds later is given by $y = 40t - 16t^2$. Find the average velocity for the time period beginning when $t = 2$ and lasting

   (i) 0.5 seconds
   (ii) 0.1 seconds
   (iii) 0.05 seconds
   (iv) 0.01 seconds

   Then, estimate the instantaneous velocity when $t = 2$.

Solution

Let $f(t) = 40t - 16t^2$. The average velocity for the time interval lasting 0.5 seconds is calculated by

$$\frac{f(2.5) - f(2)}{.5} = -32.$$

Doing this for all the values we have

(i) 0.5 seconds→ Avg Velocity is -32 ft/s

(ii) 0.1 seconds → Avg Velocity is -25.6 ft/s

(iii) 0.05 seconds → Avg Velocity is -24.8 ft/s

(iv) 0.01 seconds→ Avg Velocity is -24.16 ft/s

We can estimate the instantaneous velocity to be -24 ft/s.

6. A tank holds 1000 gallons of water, which drains from the bottom of the tank in half an hour. The values in the following table show the volume V of the water remaining in the tank (in gallons) after t minutes.

t	V
5	694
10	444
15	250
20	111
25	28
30	0

If P is the point $(15, 250)$ on the graph of V , find the slopes of the secant lines PQ when Q is a point on the graph with $t = 10$ and $t = 20$. Use these to estimate the slope of the tangent line at P by averaging the slopes of the two secant lines.

Solution

Slope of the secant line between $t = 10$ and $t = 15$:

$$\frac{250 - 444}{5} = \frac{-194}{5}$$

Slope of the secant line between $t = 15$ and $t = 20$:

$$\frac{111 - 250}{5} = \frac{-139}{5}$$

Average those two slopes:

$$\left(\frac{-194}{5} + \frac{-139}{5}\right)\frac{1}{2} = \frac{-333}{10} = -33.3\text{gal/min}$$