Know the following Rules of Logarithms.

(1) $\log_a x = y \iff a^y = x$

(2) $a^{\log_a M} = M$

(3) $\log_a a = 1$

(4) $\log_a 1 = 0$

(5) $\log_a M^r = r \log_a M$

(6) $\log_a(M \cdot N) = \log_a M + \log_a N$

(7) $\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$

(8) $\log_a M = \dfrac{\log_b M}{\log_b a}$ (Change of Base)

Avoid these Common Mistakes.

- $\log_a(M - N) = \dfrac{\log_a M}{\log_a N}$
- $\log_a(M + N) = \log_a M + \log_a N$
- $\dfrac{\log_b M}{\log_b a} = \dfrac{M}{a}$

1. Simplify.

   (a) $\log_{10} 10^2$

   (b) $\ln e^4$

   (c) $\log_2 8^{4/3}$

   (d) $\log_3 3\sqrt{3}$

   (e) $4^{\log_4 5} \cdot 5^{\log_5 4}$

2. Use the change of base formula.

   (a) Express $\log_3 2$ in terms of log base 10.

   (b) Express $\log_2 5$ in terms of the natural log.

3. Combine into a single logarithm.

   (a) $3 \log_{10} x + \log_{10} y$

   (b) $\frac{1}{2} \log_2 y - \log_2 x$

   (c) $\ln(x + 1) - \ln(x - 1)$

   (d) $\log_b 2x + 2 \log_b y + \log_b z$

   (e) $\ln 2 - \ln y + \frac{1}{2} \ln(x + y) - \frac{1}{3} \ln 27$

4. Write as a sum or difference of logarithms without any exponents.
   Hint: First factor $x^2 - y^2$ in (c) and $x^2 - 1$ in (e).

   (a) $\log_{10} \dfrac{2x^3}{y^4}$

   (b) $\ln 5x^2y^3$

   (c) $\log_2(x^2 - y^2)$

   (d) $\log_a \dfrac{1}{\sqrt{x^2 + y^2}}$

   (e) $\log_b \sqrt[3]{\dfrac{x^2 - 1}{x^4}}$