Know the following Rules of Logarithms.

(1) $\log_a x = y \iff a^y = x$

(2) $a^{\log_a M} = M$

(3) $\log_a a = 1$

(4) $\log_a 1 = 0$

(5) $\log_a M^r = r \log_a M$

(6) $\log_a(M \cdot N) = \log_a M + \log_a N$

(7) $\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$

(8) $\log_a M = \dfrac{\log_b M}{\log_b a}$ (Change of Base)

Avoid these Common Mistakes.

- $\log_a(M - N) = \dfrac{\log_a M}{\log_a N}$
- $\log_a(M + N) = \log_a M + \log_a N$
- $\dfrac{\log_b M}{\log_b a} = \dfrac{M}{a}$

1. Simplify.
   (a) $\log_{10} 10^2$ 2
   (b) $\ln e^4$ 4
   (c) $\log_2 8^{4/3}$ 4
   (d) $\log_3 3\sqrt{3}$ $\frac{3}{2}$
   (e) $4^{\log_4 5} \cdot 5^{\log_5 4}$ 20

2. Use the change of base formula.
   (a) Express $\log_3 2$ in terms of log base 10. $\frac{\log_{10} 2}{\log_{10} 3}$
   (b) Express $\log_2 5$ in terms of the natural log. $\frac{\ln 5}{\ln 2}$

3. Combine into a single logarithm.
   (a) $3\log_{10} x + \log_{10} y$ $\log_{10}(x^3 y)$
   (b) $\frac{1}{2}\log_2 y - \log_2 x$ $\log_2\left(\frac{\sqrt{y}}{x}\right)$
   (c) $\ln(x+1) - \ln(x-1)$ $\ln\left(\frac{x+1}{x-1}\right)$
   (d) $\log_b 2x + 2\log_b y + \log_b z$ $\log_b(2xy^2z)$
   (e) $\ln 2 - \ln y + \frac{1}{2}\ln(x+y) - \frac{1}{3}\ln 27$ $\ln\left(\frac{2\sqrt{x+y}}{3y}\right)$

4. Write as a sum or difference of logarithms without any exponents.
   Hint: First factor $x^2 - y^2$ in (c) and $x^2 - 1$ in (e).
   (a) $\log_{10} \dfrac{2x^3}{y^4}$ $\log_{10} 2 + 3\log_{10} x - 4\log_{10} y$

(b) $\ln 5x^2y^3$ $\quad \ln 5 + 2\ln x + 3\ln y$

(c) $\log_2(x^2 - y^2)$ $\quad \log_2(x+y) + \log_2(x-y)$

(d) $\log_a \dfrac{1}{\sqrt{x^2+y^2}}$ $\quad -\frac{1}{2}\log_a(x^2+y^2)$

(e) $\log_b \sqrt[3]{\dfrac{x^2-1}{x^4}}$ $\quad \frac{1}{3}(\log_b(x+1) + \log_b(x-1) - 4\log_b x)$