

Follow these steps to graph a rational function.

Step 1: Find all roots and their degrees. Plot the roots and note the behavior of the graph at each x -intercept. Plot the y -intercept, if there is one.

Step 2: Find all asymptotes, and list the degrees of the vertical asymptotes. Graph the asymptotes. Using their degrees and the key number method, determine the behavior of the graph at each vertical asymptote. Find all points, if any, where the horizontal or oblique asymptotes intersect the graph.

Step 3: Find the leading term and use it to determine the long-term behavior of each rational function.

Step 4: Graph the function. (The graph must be smooth.)

Graph each of the following rational functions.

$$1. f(x) = \frac{1}{(x-3)(x+3)}$$

$$2. g(x) = \frac{2}{(x+4)^2}$$

$$3. h(x) = \frac{-x}{(x-1)(x+2)}$$

$$4. r(x) = \frac{x(x-3)^2}{(x+2)(x-2)^2}$$

$$5. t(x) = \frac{x^2 + 2x + 1}{x-1}$$