Let $p(x)$ be a polynomial and let c be a number.

Remainder Theorem:
If you divide $p(x)$ by $x-c$, then the remainder will be $p(c)$.

Factor Theorem:
If $p(c)=0$, then $x-c$ is a factor of $p(x)$.
Conversely, if $x-c$ is a factor of $p(x)$, then $p(c)=0$.

Definition: c is a *root* of $p(x)$ if and only if $p(c)=0$.

1. Divide using polynomial long division. Identify the divisor, quotient, and remainder.
   (a) $\dfrac{2x^3+4x^2-5}{x+3}$ Div: $x+3$, $q(x)=2x^2-2x+6$, $r(x)=-23$
   (b) $\dfrac{2x^3-4x+7x^2+7}{x^2+2x-1}$ Div: x^2+2x+1, $q(x)=2x+3$, $r(x)=-8x+10$
   (c) $\dfrac{4x^3-2x^2-3}{2x^2-1}$ Div: $2x^2-1$, $q(x)=2x-1$, $r(x)=2x-4$
   (d) $\dfrac{x^3-5x^2+3x+7}{x-3}$ Div: $x-3$, $q(x)=x^2-2x-3$, $r(x)=-2$
   (e) $\dfrac{3x^3+5x-1}{x+1}$ Div: $x+1$, $q(x)=3x^2-3x+8$, $r(x)=-0$
   (f) $\dfrac{4x^3-8x^2-x+5}{2x-1}$ Div: $2x-1$, $q(x)=2x^2-3x-2$, $r(x)=3$

2. What is the remainder if you divide each of the following $p(x)$ by $x-2$? Is $x-2$ a factor of $p(x)$?
   Hint: Use the Remainder and Factor Theorems.
   (a) $p(x)=-2x^3+5x-1$ No, $r(x)=-7$
   (b) $p(x)=3x^2-x^5+7x+6$ Yes, $r(x)=0$
   (c) $p(x)=x^4+6x^3-x^2+10$ No, $r(x)=70$

3. Find all roots of $p(x)$.
   (a) $p(x)=x^3-19x-30$, given that 5 is a root Roots at $x=-3,-2,5$
   (b) $p(x)=x^3-6x+4x^2-24$, given that -4 is a root Roots at $x=-4,\pm\sqrt{6}$
   (c) $p(x)=8x^3-10x^2-x+3$, given that $\frac{3}{4}$ is a root Roots at $x=\frac{3}{4},1,-\frac{1}{2}$
   (d) $p(x)=x^3-6x^2+11x-6$ Roots at $x=1,2,3$
   (e) $p(x)=x^4+x^3-7x^2-x+6$ Roots at $x=1,-1,2,-3$

   Hint: Use the theorems and definition to help you find at least one root for (d),(e).