Know the **quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

which yields the solutions to the equation $ax^2 + bx + c = 0$.

The **discriminant**, $b^2 - 4ac$, tells us how many *real solutions* there are:

(i) If $b^2 - 4ac > 0$, there are two real solutions.

(ii) If $b^2 - 4ac = 0$, there is one real solution.

(iii) If $b^2 - 4ac < 0$, there are no real solutions.

1. For each of the following quadratic functions
   - use the discriminant to determine the number of $x-$intercepts
   - find the $x-$intercepts (if applicable) by solving $ax^2 + bx + c = 0$ using a method of your choice
   - use the method of completing the square to rewrite the function in the form
     $$a(x-h)^2 + k$$
     where (h, k) is the vertex of the parabola and $x = h$ is the axis of symmetry
   - find the $y-$intercept
   - graph the parabola

   (a) $f(x) = x^2 + 6x + 9$
      i. 1 x-intercept
      ii. $x = -3$
      iii. $(x+3)^2 = f(x)$
      iv. $(0, 9)$

   (b) $f(x) = x^2 - 2x + 2$
      i. no x-intercepts
      ii. None
      iii. $(x-1)^2 + 1 = f(x)$
      iv. $(0, 2)$

   (c) $f(x) = -2x^2 - 4x + 2$
      i. 2 x-intercepts

ii. $x = -1 \pm \sqrt{2}$

iii. $-2(x+1)^2 + 4 = f(x)$

iv. $(0, 2)$

(d) $f(x) = -x^2 - x + 2$

i. 2 x-intercepts

ii. $x = -2, x = 1$

iii. $-(x + \frac{1}{2})^2 + \frac{9}{4} = f(x)$

iv. $(0, 2)$

2. For each of the following optimization problems

- set up a quadratic function
- find the vertex of the parabola
- answer the question

(a) A rectangular garden has a perimeter of 16 meters. What is its largest possible area?

i. $A(w) = -w^2 + 8w$

ii. $(4, 16)$

iii. Largest possible area is $16m^2$.

(b) The sum of two numbers is 10. What is the largest possible product of the two numbers?

i. $P(x) = -x^2 + 10x$

ii. $(5, 25)$

iii. The largest possible product is 25.

(c) The difference of two numbers is 2. What is the smallest possible product of the two numbers?

i. $P(x) = x^2 + 2x$

ii. $(-1, -1)$

iii. The smallest possible product is -1.