

1. Complete the square for the following expressions.

$$(a) \quad x^2 - 6x + 15 \qquad (x - 3)^2 + 6$$

$$(b) \quad x^2 + 2x - 8 \qquad (x + 1)^2 - 9$$

$$(c) \quad 4x^2 - 8x + 3 \qquad 4(x - 1)^2 - 1$$

2. Complete the square for the following equations. (Bonus: Solve the equations.)

$$(a) \quad x^2 - 8x + 12 = 0 \qquad (x - 4)^2 = 4 \qquad \text{Solutions: } x = 2 \text{ or } x = 6$$

$$(b) \quad x^2 + 3x - 6 = 0 \qquad \left(x + \frac{3}{2}\right)^2 = \frac{33}{4} \qquad \text{Solutions: } x = -\frac{3}{2} \pm \frac{\sqrt{33}}{2}$$

$$(c) \quad -3x^2 - 6x + 15 = 0 \qquad (x + 1)^2 = 6 \qquad \text{Solutions: } x = -1 \pm \sqrt{6}$$

$$(d) \quad x^2 + 3x = 0 \qquad \left(x + \frac{3}{2}\right)^2 = \frac{9}{4} \qquad \text{Solutions: } x = 0 \text{ or } x = -3$$

$$(e) \quad x^2 + x = 0 \qquad \left(x - \frac{1}{2}\right)^2 = \frac{1}{4} \qquad \text{Solutions: } x = 0 \text{ or } x = 1$$

$$(f) \quad 2x^2 - 24x = -71 \qquad 2(x - 6)^2 = 1 \qquad \text{Solutions: } x = 6 \pm \frac{\sqrt{2}}{2}$$

3. Find the center and radius of the circles represented by the following equations.

Hint: Use the method of completing the square to rewrite the equation in the form

$$(x - h)^2 + (y - k)^2 = r^2$$

where (h, k) is the center and r is the radius.

$$(a) \quad x^2 + y^2 - 6x - 8y = 0 \\ (x - 3)^2 + (y - 4)^2 = 5^2 \qquad \text{center: } (3, 4), \text{ radius: } 5$$

$$(b) \quad x^2 + y^2 - 4x - 2y = 11 \\ (x - 2)^2 + (y - 1)^2 = 4^2 \qquad \text{center: } (2, 1), \text{ radius: } 4$$

$$(c) \quad 2x^2 + 2y^2 + 4x + 8y - 20 = 0 \\ (x + 1)^2 + (y + 2)^2 = \sqrt{15}^2 \qquad \text{center: } (-1, -2), \text{ radius: } \sqrt{15}$$