1. Write the sum $\sum_{i=1}^{4} \frac{i-3}{i}$ without sigma notation, then evaluate it.

Solution

$$\frac{1-3}{1}+\frac{2-3}{2}+\frac{3-3}{3}+\frac{4-3}{4}=-2-\frac{1}{2}+0+\frac{1}{4}=-\frac{9}{4}$$

2. Approximate the area under the graph of $f(x)=2x^2$ and above the x-axis from $x=0$ to $x=4$ using $n=4$ and the following methods.

   (a) Using left endpoints.

   Solution

   Since $n = 4$ then $\Delta x = \frac{4-0}{4} = 1$, thus the intervals are $[0,1], [1,2], [2,3], [3,4]$. Using left endpoints, the approximation is

   $$f(0)\cdot 1+f(1)\cdot 1+f(2)\cdot 1+f(3)\cdot 1=28$$

   (b) Using right endpoints.

   Solution

   Using the same intervals as above, we have the right endpoint approximation is:

   $$f(1)+f(2)+f(3)+f(4)=60$$

   (c) Using the midpoints.

   Solution

   Using the same intervals as above, we have the midpoint approximation is:

   $$f(\frac{1}{2})+f(\frac{3}{2})+f(\frac{5}{2})+f(\frac{7}{2})=42$$

3. Express the limit as a definite integral on the interval $[2,5]$. (Do not solve the integral).

$$\lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{x_i^2-2}\Delta x$$

Solution

$$\int_2^5 \sqrt{x^2-2}dx$$

4. Suppose that f and g are integrable. Find the integrals below using the following information:

$$\int_1^9 f(x)dx = -2, \; \int_7^9 f(x)dx = 10, \; \int_7^9 g(x)dx = 3$$

(a) $\int_9^7 [f(x) - g(x)]dx$

Solution

$$\begin{aligned}\int_9^7 [f(x) - g(x)]dx &= -\int_7^9 [f(x) - g(x)]dx \\ &= -[\int_7^9 f(x)dx - \int_7^9 g(x)dx] \\ &= -\int_7^9 f(x)dx + \int_7^9 g(x)dx \\ &= -10 + 3 = -7\end{aligned}$$

(b) $\int_1^7 f(x)dx$

Solution

$$\begin{aligned}\int_1^7 f(x)dx &= \int_1^9 f(x)dx - \int_7^9 f(x)dx \\ &= -2 - 3 = -5\end{aligned}$$

(c) $\int_7^9 [3g(x) + 2f(x)]dx$

Solution

$$\begin{aligned}\int_7^9 [3g(x) + 2f(x)]dx &= 3\int_7^9 g(x)dx + 2\int_7^9 f(x)dx \\ &= 3 \cdot 3 + 2 \cdot 10 = 29\end{aligned}$$

5. Evaluate the following integrals.

(a) $\int_{-2}^2 (x^3 - 2x + 1)dx$

Solution

$$\int_{-2}^{2}(x^3-2x+1)dx = \int_{-2}^{2} x^3dx - \int_{-2}^{2} 2xdx + \int_{-2}^{2} 1dx$$

Notice either by computing the Reimann sum or looking at a graph that $\int_{-2}^{2} x^3dx = 0$ as well as $\int_{-2}^{2} 2xdx = 0$. Thus

$$\begin{aligned}\int_{-2}^{2}(x^3-2x+1)dx &= \int_{-} 2^2 1dx \\ &= \lim_{n\to\infty} \sum i = 1^n 1 \cdot \frac{4}{n} \\ &= \lim_{n\to\infty} \frac{4}{n} \sum i = 1^n 1 \\ &= \lim_{n\to\infty} \frac{4}{n} \cdot n \\ &= \lim_{n\to\infty} 4 = 4\end{aligned}$$

(b) $\int_0^4 (3-2x)dx$

Solution

$$\Delta x = \frac{4}{n} = 1 \qquad x_i = 0 + \frac{4}{n}i = \frac{4i}{n}$$

$$\begin{aligned}\int_0^4 (3-2x)dx &= \lim_{n\to\infty} \sum_{i=1}^{n} \left(3 - 2(\frac{4i}{n})\right) \cdot \frac{4}{n} \\ &= \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} (3 - \frac{8i}{n}) \\ &= \lim_{n\to\infty} \frac{4}{n} \left[\sum_{i=1}^{n} 3 - \sum i = 1^n \frac{8i}{n}\right] \\ &= \lim_{n\to\infty} \frac{4}{n} \left[3n - \frac{8}{n} \sum i = 1^n i\right] \\ &= \lim_{n\to\infty} \frac{4}{n} \left[3n - \frac{8}{n}\frac{n(n+1)}{2}\right] \\ &= \lim_{n\to\infty} 12 - \frac{16n(n+1)}{n^2} \\ &= 12 - 16 = -4\end{aligned}$$