1. The position of a particle moving along a coordinate axis is given by $s(t) = t^3 - 9t^2 + 24t + 4$, where t is time, in seconds.

   (a) Find the velocity of the particle, $v(t)$.

   Solution

   The derivative of the position function is the velocity function.

   $$s'(t) = v(t) = 3t^2 - 18t + 24$$

   (b) At what time(s) is the particle at rest?

   Solution

   The particle is at rest when the velocity is 0.

   $$\begin{aligned} 0 &= 3t^2 - 18t + 24 \\ 0 &= t^2 - 6t + 8 \\ 0 &= (t-2)(t-4) \\ t &= \{2, 4\} \end{aligned}$$

   The particle is at rest at 2 and 4 seconds.

   (c) On what time intervals is the particle moving from left to right? From right to left?

   Solution

   The particle is moving left to right (forward) when the velocity is positive, and moving right to left when the velocity is negative. Since $v(1), v(5) > 0$ and $v(3) < 0$ then the particle is moving left to right on $[0, 2) \cup (4, \infty)$, and it is moving right to left on $(2, 4)$.

   (d) When is the item speeding up? When is it slowing down?

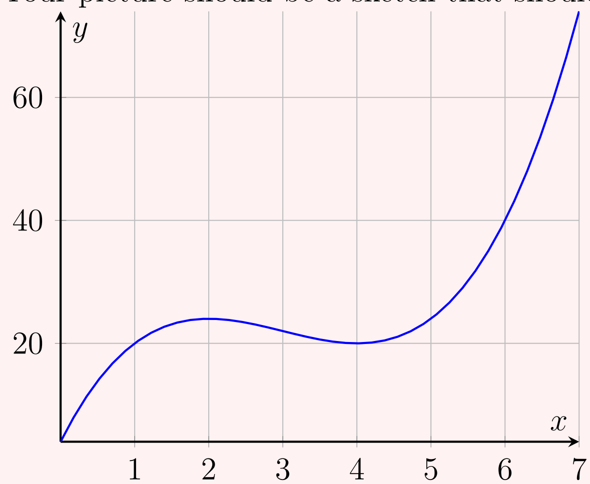
   Solution

   The particle is speeding up when the sign of the velocity matches the sign of the acceleration, and it is slowing down when the velocity and acceleration have different signs.
   $a(t) = v'(t) = 6t - 18$, which is positive on $(3, \infty)$ and negative on $(0, 3)$. This implies that the particle is speeding up on $(2, 3) \cup (4, \infty)$ and slowing down on $(0, 2) \cup (3, 4)$.

(e) Use the information obtained to sketch the path of the particle along a coordinate axis.

Solution

Your picture should be a sketch that should have the same general shape.



2. The radius of a sphere is increasing at a rate of 2 cm/min . At what rate is the surface area inreasing when the radius is 10 cm?

Solution

Let r denote the radius and A denote the surface area. Then we know that $\frac{dr}{dt} = 2$, and we are interested in $\frac{dA}{dt}$ when $r = 10$. We have that $A = 4\pi r^2$. Differentiating both sides with respect to t we get

$$\begin{aligned} \frac{dA}{dt} &= 4\pi(2r \cdot \frac{dr}{dt}) \\ &= 8\pi r \frac{dr}{dt} \end{aligned}$$

when $r = 10$ we have that

$$\begin{aligned} \frac{dA}{dt} &= 8\pi(10)(2) \\ &= 160\pi \end{aligned}$$

So the surface area increases at a rate of 160π cm^2/min when the radius is 10 cm.

3. Water is poured into a conical container at the rate of 10 cm^3/sec. The cone points directly down, and it has a height of 30cm and a base radius of 10cm. How fast is the water level rising when the water is4cmdeep (at its deepest point)?

Solution

Let V represent the volume of the water in the container. Then $V = \frac{\pi}{3}r^2 * h$ where h is the height of the water at the deepest point, and r is the radius of the container at that height. Using similar triangles and the size of the container, $\frac{h}{r} = \frac{30}{10}$, so $\frac{1}{3}h = r$. We can substitute this into the equation for the volume of the water in the container to get

$$V = \frac{\pi}{27}h^3$$

We are looking for $\frac{dh}{dt}$ when $h = 4$ and we know that $\frac{dV}{dt} = 10$.
We will differentiate the volume formula with respect to t.

$$\frac{dV}{dt} = \frac{\pi}{27}(3h^2 \cdot \frac{dh}{dt})$$
$$\frac{dV}{dt} = \frac{\pi}{9}h^2\frac{dh}{dt}$$
$$10 = \frac{\pi}{9}(4)^2\frac{dh}{dt}$$
$$10 = \frac{16\pi}{9}\frac{dh}{dt}$$
$$\frac{90}{16\pi} = \frac{dh}{dt}$$
$$\frac{45}{8\pi} = \frac{dh}{dt}$$

The water level is rising at a rate of $\frac{45}{8\pi}$ cm/sec when the water is 4 feet deep.

4. A kite 100 ft above the ground moves horizontally at a speed of 8 ft/s. At what rate is the angle between the string and the horizontal decreasing when 200 ft of string have been let out?

Solution

Let θ represent the angle between the ground (horizontal) and the string, h represent the horizontal length, and s represent the length of the string. We know that $\frac{dh}{dt} = 8$ and we are looking for $\frac{d\theta}{dt}$ when $s = 200$.
Using what we know about trig, $\sin(\theta) = \frac{100}{s}$ and $\cos(\theta) = \frac{h}{s}$ Let us choose to differentiate $\sin(\theta)$ with respect to t. (Note: This is not the only option.)

$$\frac{d}{dt}(\sin(\theta) = \frac{d}{dt}\left(\frac{100}{s}\right)$$
$$\cos(\theta)\frac{d\theta}{dt} = -\frac{100}{s^2} \cdot \frac{ds}{dt}$$
$$\frac{h}{s}\frac{d\theta}{dt} = -\frac{100}{s^2} \cdot \frac{ds}{dt}$$
$$\frac{d\theta}{dt} = -\frac{100}{sh} \cdot \frac{ds}{dt}$$

We need to determine $\frac{ds}{dt}$. By Pythagorean Theorem $s^2 = h^2 + 100^2$. Differentiate with respect to t to determine $\frac{ds}{dt}$.

$$2s\frac{ds}{dt} = 2h\frac{dh}{dt}$$
$$\frac{ds}{dt} = \frac{h}{s}\frac{dh}{dt}$$

Putting this together with $\frac{d\theta}{dt}$, we have

$$\frac{d\theta}{dt} = -\frac{100}{sh} \cdot \frac{h}{s}\frac{dh}{dt} = -\frac{100}{s^2}\frac{dh}{dt}$$

Using the fact that $\frac{dh}{dt} = 8$, and evaluating when $s = 200$, we have

$$\frac{d\theta}{dt} = -\frac{100}{(200)^2} \cdot 8 = -0.02$$

So the angle is decreasing at a rate of -0.02 rad/sec.

5. Let $f(x) = \sqrt{x}$. If $a = 1$ and $dx = \Delta x = \frac{1}{10}$, what are Δy and dy?

Solution

$$\begin{aligned}\Delta y &= f(x + \Delta x) - f(x)\\ &= \sqrt{1.1} - \sqrt{1}\\ &\approx 0.0488088\end{aligned}$$

$$\begin{aligned}dy &= f'(x)dx\\ &= \frac{1}{2\sqrt{x}}\frac{1}{10}\\ &= \frac{1}{20}\\ &= 0.05\end{aligned}$$

6. Find the differential dy and evaluate dy when $x = \frac{\pi}{4}$ and $dx = -0.1$.

$$y = \tan x$$

Solution

$$\begin{aligned}dy &= \sec^2(x)dx\\ dy &= \sec^2(\frac{pi}{4}) - 0.1\\ dy &= \sqrt{2}^2(-0.1)\\ dy &= -0.2\end{aligned}$$