

1. Find the following limits.

(a) $\lim_{x \rightarrow 3} \frac{1}{x-3}$

(b) $\lim_{x \rightarrow 0} \frac{x^3 - 4x}{2x + 1}$

(c) $\lim_{x \rightarrow 3} \frac{\frac{1}{x} - \frac{1}{3}}{x-3}$

(d) $\lim_{x \rightarrow -2} \frac{1}{x^2 - 4}$

(e) $\lim_{x \rightarrow 4} \frac{x^2 - 7x + 12}{2x - 8}$

2. Find $\lim_{x \rightarrow 0^-} x^3 \cos\left(\frac{2}{x}\right)$. (Hint: Squeeze theorem.)

3. Determine where the following function is continuous, and state why.

$$f(x) = \frac{1}{\sqrt{x+3}} - \frac{x^2 - 1}{x - 1} - 4x^2$$

4. Is the following function continuous at $x = 0$? $f(x) = \begin{cases} \frac{x-6}{x-3} & x < 0 \\ 2 & 0 \leq x = 0 \\ \sqrt{4+x^2} & x > 0 \end{cases}$

5. Use the precise definition of a limit (ϵ and δ) to prove the following limit.

$$\lim_{x \rightarrow 3} (3x + 5) = 35$$

6. Find where the following function is discontinuous, and state the types of discontinuities.

$$f(x) = \frac{x^2 - 3x + 2}{x^2 - x - 6}$$

7. Use the Intermediate Value Theorem to show that $h(x) = 4x^2 - 29x^2 + 25$ has a real root in the interval $(-3, -2)$.