Set Theory: Operations

1. Let $U = \{a, b, c, d, e, f, u, v, w, x, y, z\}$, $X = \{a, e, u, y\}$, $Y = \{a, b, d, y, z\}$, and $Z = \{c, f, u, v, y\}$.

   (a) Find $X \cap Y$.
   $\{a, y\}$

   (b) Find $X \cup Y$.
   $\{a, e, u, y, b, d, z\}$

   (c) Find $Y - X$.
   $\{b, d, z\}$

   (d) Find $(X \cap Y)' \cup Z$.
   $\{b, c, d, e, f, u, v, w, x, y, z\}$

   (e) Find $(X' \cup Y') \cap Z$.
   $\{\}$

   (f) Find $Y - (X \cup Z)'$.
   $\{a, y\}$

2. Let A and B represent any two sets. Determine if the following statements are *always true*, *not always true*, or *always false*.

   (a) $(A \cap B) \subseteq A$
   Always true

   (b) $A \subset B$
   Not always true

   (c) $(A \cup B) \subset A$
   Not always true

   (d) $n(A \cup B) = n(A) + n(B)$
   Not always true

   (e) $n(A - B) = n(A) - n(B)$
   Not always true

   (f) $n(A' \cap B') = n((A \cup B)')$
   Always true

3. Let $A = \{0, 1, 2, 3\}$ and $B = \{2, 4\}$. Find $A \times B$.
   $\{(0, 2), (0, 4), (1, 2), (1, 4), (2, 2), (2, 4), (3, 2), (3, 4)\}$

4. Let $n(S) = 12$ and $n(T) = 10$. Determine $n(S \times T)$.
   $n(S \times T) = 120$

5. A co-ed soccer team needs to elect two captains. They want one captain to be a female, and one a male. The team has 7 females and 9 males. How many different pairs of captains can they have?
   $n(Pairs) = 7 \times 9 = 63$

6. The operations addition and multiplication, when applied to numbers, have some convenient properties. Let a, b, c be real numbers. Then the following properties hold.
   - Commutative property of addition: $a + b = b + a$
   - Associative property of addition: $(a + b) + c = a + (b + c)$
   - Identity element for addition, 0: $a + 0 = a$
   - Distributive property of multiplication over addition: $a(b + c) = ab + ac$

   Use examples, venn diagrams, and mathematical reasoning to answer the following questions.
   While general solutions are given, it is advised that you explore the solutions on your own. Draw venn diagrams to help you visualize the operations.

   (a) Do you think multiplication is commutative and associative? Try some examples and discuss why or why not you think so.
   Yes

   (b) Does multiplication have an identity element?
   Yes, 1. $a \times 1 = a$ for every real number a.

   (c) Is set union commutative? What about set intersection?
   Yes, $A \cup B = B \cup A$

   (d) Is set union associative? What about set intersection?
   Yes and Yes

   (e) Is there an identity element for set union? If so, what is it? What about for set intersection?
   The identity for set union is $\{\}$. The identity for set intersection is U.

   (f) Is set intersection distributive over set union? Is set union distributive over set intersection?
   Yes