

For the following functions ...

- List any roots and the behavior of the graph at the roots.
- List all asymptotes, and the degree of all vertical asymptote.
- Find any points where the graph intersects a horizontal or oblique asymptote.
- Find the leading term and use it to determine the long term behavior.
- Label your axes

1.  $f(x) = \frac{x^2 + 3x + 2}{x - 1}$

Roots:  $x = -2, x = -1$ , both degree 1 (crosses)

VA:  $x = 1$ , degree 1

OA:  $y = x + 4$ , doesn't intersect

Leading term:  $x$ ,  $f(x) \rightarrow \infty$  as  $x \rightarrow \infty$ ,  $f(x) \rightarrow -\infty$  as  $x \rightarrow -\infty$ .

2.  $g(x) = \frac{3}{x^2 - 5x + 6}$

Roots: None

VA:  $x = 2, x = 3$ , both degree 1

HA:  $y = 0$ , intersects at roots

Leading term:  $\frac{3}{x^2}$ ,  $f(x) \rightarrow 0$  as  $x \rightarrow \pm\infty$ .

3.  $f(x) = (x - 3)(x + 2)(x)^2$

Roots:  $x = 3, x = -2$ , both degree 1 (crosses),  $x = 0$ , degree 2 (touches)

VA: None

OA/HA: None

Leading term:  $x^4$ ,  $f(x) \rightarrow \infty$  as  $x \rightarrow \pm\infty$ .

4.  $f(x) = \frac{x^2 - 16}{x}$

Roots:  $x = -4, x = 4$ , both degree 1 (cross)

VA:  $x = 0$ , degree 1

OA:  $y = x$ , doesn't intersect

Leading term:  $x$ ,  $f(x) \rightarrow \infty$  as  $x \rightarrow \infty$ ,  $f(x) \rightarrow -\infty$  as  $x \rightarrow -\infty$ .

5.  $h(x) = (x + 4)^3(x - 1)^2$

Roots:  $x = -4$ , degree 3 (cross),  $x = 1$ , degree 2 (touches)

VA: None

OA/HA: None

Leading term:  $x^5$ ,  $f(x) \rightarrow \infty$  as  $x \rightarrow \infty$ ,  $f(x) \rightarrow -\infty$  as  $x \rightarrow -\infty$ .

6.  $g(x) = (x + 2)(x)(2 - x)$

Roots:  $x = -2, x = 0, x = 2$ , all degree 1 (crosses)

VA: None

OA/HA: None

Leading term:  $-x^3$ ,  $f(x) \rightarrow -\infty$  as  $x \rightarrow \infty$ ,  $f(x) \rightarrow \infty$  as  $x \rightarrow -\infty$ .

$$7. f(x) = \frac{(x-1)(x+1)}{(x-3)(x+3)}$$

Roots:  $x = 1, x = -1$ , both degree 1 (crosses)

VA:  $x = 3, x = -3$ , both degree 1

HA:  $y = 1$ , doesn't intersect

Leading term: 1,  $f(x) \rightarrow 1$  as  $x \rightarrow \pm\infty$

$$8. q(x) = \frac{1}{x^2}$$

Roots: None

VA:  $x = 0$ , degree 2

HA:  $y = 0$ , doesn't intersect

Leading term:  $\frac{1}{x^2}$ ,  $f(x) \rightarrow 0$  as  $x \rightarrow \pm\infty$ .