1. We say that a function f is the *inverse* of a function g if

$$(f \circ g)(x) = x \text{ and } (g \circ f)(x) = x.$$

Verify that the given functions are inverses of each other.

(a) $f(x) = 2x - 8, \quad g(x) = \frac{1}{2}x + 4$
Solution:

$$f(g(x)) = f(\frac{1}{2}x + 4) = 2(\frac{1}{2}x + 4) - 8 = x + 8 - 8 = x$$

$$g(f(x)) = g(2x - 8) = \frac{1}{2}(2x - 8) + 4 = x - 4 + 4 = x$$

(b) $f(x) = \sqrt[3]{x} + 1, \quad g(x) = (x - 1)^3$

(c) $f(x) = \frac{1}{x + 1}, \quad g(x) = \frac{1}{x} - 1$

2. For each f, compute f^{-1}. Then find the range of f by finding the domain of f^{-1}.

(a) $f(x) = 4x - 1$
Solution: $f^{-1}(x) = \frac{1}{4}x + \frac{1}{4}$ Range of $f : \mathbb{R}$.

(b) $f(x) = 2x^3 - 1$
Solution: $f^{-1}(x) = \sqrt[3]{\frac{x}{2} + \frac{1}{2}}$ Range of $f : \mathbb{R}$

(c) $f(x) = \frac{2}{x - 3}$
Solution: $f^{-1}(x) = \frac{2}{x} + 3$ Range of $f : \mathbb{R} \setminus \{0\}$.

(d) $f(x) = \frac{x - 5}{x + 2}$
Solution: $f^{-1}(x) = \frac{5 + 2x}{1 - x}$ Range of $f : \mathbb{R} \setminus \{1\}$.

3. Determine whether each function is one-to-one.

(a) $f(x) = 2$ No
(b) $f(x) = 3x - 1$ Yes
(c) $f(x) = x^2$ No
(d) $f(x) = x^3$ Yes
(e) $f(x) = \sqrt{x}$ No
(f) $f(x) = \sqrt[3]{x}$ Yes
(g) $f(x) = |x|$ No
(h) $f(x) = \frac{1}{x}$ Yes
(i) $f(x) = \frac{1}{x^2}$ No

4. For each function in #3 that was one-to-one, compute its inverse.

(b) $f^{-1}(x) = \frac{x+1}{3}$
(d) $f^{-1}(x) = \sqrt[3]{x}$
(f) $f^{-1}(x) = x^3$
(h) $^{-1}(x) = \frac{1}{x}$

5. For each function in #3 that was <u>not</u> one-to-one, state the largest subset of its domain for which the function would be one-to-one.

   (a) Any single point

   (c) $[0, \infty)$

   (e) $[0, \infty)$

   (g) $[0, \infty)$

   (i) $[0, \infty)$